FREQUENCY DOMAIN BASED FEED FORWARD TUNING FOR FRICTION COMPENSATION

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ABSTRACT

In high precision motion control, performance is often limited by the presence of nonlinearities. In this study, the presence of nonlinear influences in a high precision transmission electron microscope stage is investigated using broadband multisine signals. These measurements yield the nature and level of nonlinearities as well as the best linear approximation of the dynamics. By quantitatively measuring the level of nonlinear influences, this method indicates the relevance of improved modeling. Next, the nonlinear influences are modeled explicitly by measuring the higher order sinusoidal input describing functions (HOSIDF) of the system which describe the 'direct' response of the system at the input frequency as well as at harmonics of the input frequency. Application of this technique yields a structured way to design Coulomb friction feed forward in the presence of nonlinearities. This procedure linearizes the input-output dynamics by applying feed forward and measuring the HOSIDFs which indicate the remaining nonlinear effects. Application of this technique yields a structured way to design feed forward in the presence of nonlinearities.

INTRODUCTION

When identifying and controlling (mechanical) systems, a linear model structure is often assumed. If nonlinear influences are small, such assumptions may be justified. In order to draw conclusions about nonlinear influences (type and magnitude) the authors in [1, 2, 3] present a multisine based, frequency domain identification approach. This method yields both the Best Linear Approximation (BLA) of the systems dynamics and the magnitude and type of nonlinearities present. To further quantify the nonlinear influences the authors in [4, 5, 6] present a nonparametric modeling technique referred to as Higher Order Sinusoidal Input Describing Functions (HOSIDF). This technique describes the response of a system by relating the magnitude and phase of the harmonics of a sinusoidal input, in the output signal due to nonlinear influences. Finally, this study extends the concept behind the HOSIDFs to optimal feed forward design for a friction dominated motion system.

In the first section two experimental set-ups used in this study are introduced. Next, the multisine based approach is used to measure the BLA of the dynamics of an industrial high precision motion stage. This method emphasizes the relevance of improved modeling by **detecting** the magnitude and type of nonlinearities present. Moreover, the HOSIDFs of the same system are measured to further **quantify** the nonlinear behaviour. Finally, the concept behind the HOSIDFs is used to design optimal feed forward **control** for a 4th order motion system with Coulomb friction, minimizing the amount of nonlinear influence and largely improving the systems performance.

EXPERIMENTAL SET-UP

In this paper two experimental set-ups are considered. First, an industrial motion stage used to control the sample position in a transmission electron microscope (TEM) is used to demonstrate the different measurement techniques. This application is selected since the performance required in this application is particularly high. The high reproducibility and resolution required impose strong requirements on the control loop

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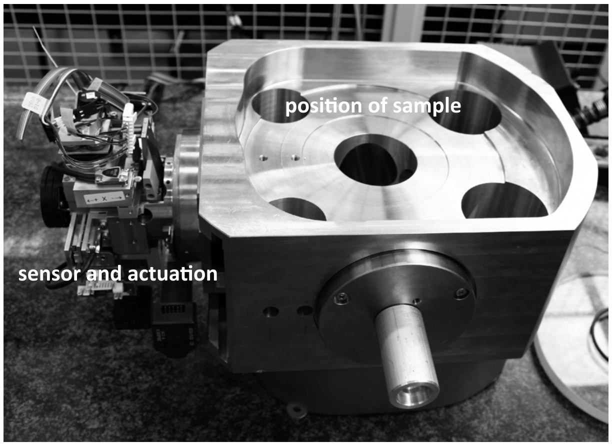

FIGURE 1. Experimental set-up (I): High precision TEM motion stage.

used to control the system. Second, a 4th order mass, spring, damper system with Coulomb friction is used to illustrate the usage of the measured nonlinear response in feed forward tuning. In the sequel the experimental set-ups are discussed in detail.

(I) Transmission electron microscope stage

Figure 1 depicts the TEM stage, removed from the TEM. The stage is used as a SISO system driven by a Maxon DC motor, with the engine voltage as input and the position of the stage as an output. The application of the excitation signal and measurement of the response is performed using a SigLab 20-42 dynamics analyzer providing 90 dB aliasing protection. The sample position is measured using a MicroE Mercury 3500 encoder with a sensor accuracy of 5 nm.

(II) 4th order mechanical system with friction

Figure 2 shows a 4th order mass, spring, damper system with Coulomb friction applied at the motor side. The system consists of two rotating masses connected by a torsional element. The friction is applied by applying a constant normal force acting through a vertical guidance system. The contact between the rotating mass and the friction element consists of two cylindrical elements resulting in an approximate single point contact. The constant normal force, combined with the single point of steel to steel contact results in a close approximation of Coulomb friction. The rotation of both masses is measured using an optical encoder with an accuracy of 500 increments per rotation and the system is driven by a Maxon DC motor. The engine voltage is the input of the system, while the rotation of the mass on the motor side (right side in Figure 2) is the output.

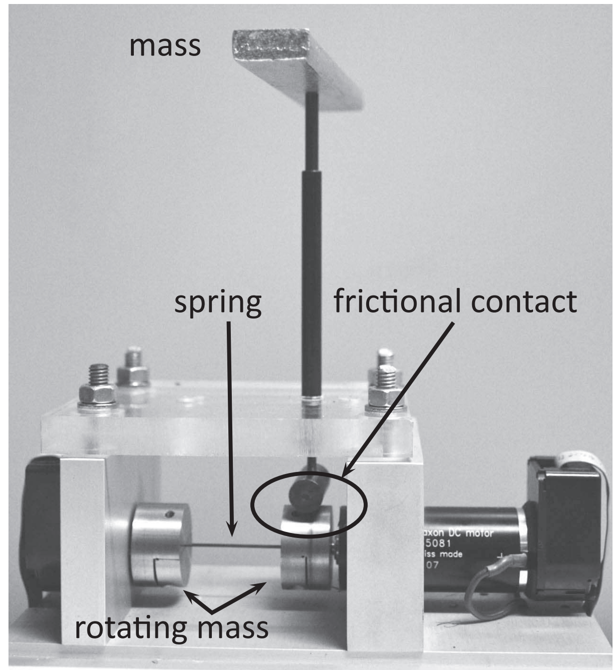

FIGURE 2. Experimental set-up (II): 4th order system with Coulomb friction.

DETECTING NONLINEARITIES

Methodology

In this section a multisine based method is introduced that allows measurements of both the best linear approximation of a system under test and the magnitude and type of nonlinearities present [1, 2, 3]. In general, output spectra Y are composed of the harmonic content generated from the input U by the best linear approximation $Y_{BLA} = H_{BLA}U$, stochastic disturbances (noise) Y_{noise} and disturbances generated by nonlinear influences Y_S [1]:

$$Y = Y_{BLA} + Y_S + Y_{noise} \tag{1}$$

Using signals with specific spectra and phase distributions called random odd multisines, the amount of information about nonlinear influences obtained from measurements in maximized. The mth realization of a random odd multisine is defined as:

$$u^{[m]}(t) = \sum_{n=1}^{N} \alpha_n \sin\left(2\pi f_n\, t + \varphi_n^{[m]}\right), \tag{2}$$

with $\alpha_n \in \mathbb{R}_{\geq 0}$ possibly different for various frequencies, but constant over different real-

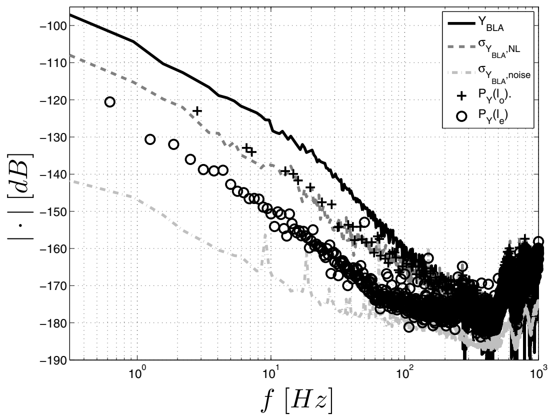

FIGURE 3. Output spectrum of a typical multisine experiment, showing the effects of nonlinearities ($\sigma_{Y_{BLA},NL}$) and noise ($\sigma_{Y_{BLA},noise}$) as variances on the best linear approximation of the spectrum ($u_{rms} = 3.72$ V).

izations. The phases $\varphi_n^{[m]}$ are randomly selected from the interval $[-\pi\ \pi)$ for each realization and the odd harmonic frequency lines $f_n \in \mathbb{F}_o = \{(2k-1)f_0^{\mathfrak{b}}|\ k \in \mathbb{N}_1,\ f_0^{\mathfrak{b}} \in \mathbb{R}_{>0}\}$ are selected identically for all realizations. The random odd multisine is particularly useful for detection of nonlinear effects when a frequency line is removed randomly every 5 odd frequency lines [2]. For convenience the same spectral lines are removed for all realizations.

When using odd random multisines, there are two ways to detect nonlinearities in the spectral representation of the measured response. First of all, energy may appear on non-excited lines in the output spectrum. Secondly, a variance, larger than to be expected based on stochastic distortions, is observed on the measured output spectrum when using different realizations of the multisine. To detect energy at non excited frequency lines, a single measurement suffices classifying the type of nonlinear effects since energy may appear on non-excited odd or even lines. However, to obtain the variance on the output spectrum at excited lines due to noise and nonlinearities separately, multiple experiments with different realizations of the excitation signal are required. In this paper both methods are combined to both quantify the extend of nonlinear effects and classify them.

First, consider the excited spectral lines in the input signal. Performing M experiments with M re-

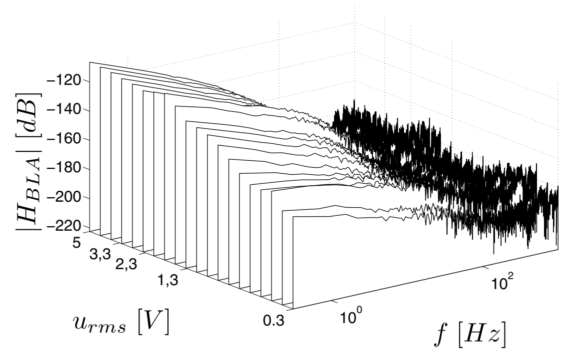

FIGURE 4. Best linear approximation H_{BLA} of the systems transfer function measured using the broadband excitation approach, as a function of input amplitude and frequency.

alizations of P periods the excitation signal yields $M \times P$ input and output time series and their corresponding spectra $U(\omega)$ and $Y(\omega)^{[m]}$. Averaging over multiple periods of the same realization yields the average spectrum and the variance on this spectrum due to stochastic distortions $\sigma^{2[m]}_{\bar{Y},noise}$, but not that due to nonlinear effects. Next, calculating the average spectrum over different realizations yields the best linear approximation Y_{BLA} of the true spectrum and the variance on this averaged spectrum due to nonlinearities $\sigma^2_{Y_{BLA},NL}$. Second, consider the response at odd (o) and even (e) non-excited lines $\mathcal{P}(\ell_{o/e})$. The spectrum has random phase at these lines, hence calculating the variance over the different measured spectra yields the average power that occurs at these frequencies. This yields a measure for nonlinear behaviour as well.

Application
The BLA and the level of nonlinearities in the industrial high precision stage are measured using the described procedure. Measurements are performed with measurement frequency of $f_s = 2560$ Hz and a block length of $N_{block} = 8192$ measurement points. This yields in a base frequency of the random odd multisine of $f_0^{\mathfrak{b}} = \frac{f_s}{N_{block}} = 0.3125$ Hz. Sufficient waiting time is allowed to assure that transient phenomena have damped out, avoiding leakage phenomena and no windowing is applied.

$M = 10$ realizations of the odd random multisine have been generated and the response has been measured for $P = 10$ periods. Furthermore, this experiment is repeated for 20 different rms values

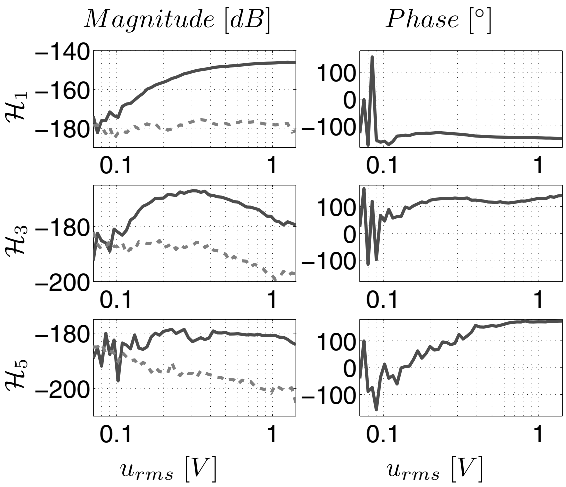

FIGURE 5. HOSIDFs at 20 Hz calculated from 10 experiment per amplitude. Mean value ($\bar{\mathcal{H}}_k$) of the HOSIDF (–) and standard deviation $\varsigma_{\bar{\mathcal{H}}_k}$ (– –).

of the multisines, logarithmically scaled between 0.3 V and 5.0 V. A typical output spectrum is depicted in Figure 3. The best linear approximation of the systems dynamics is depicted in Figure 4 as a function of both frequency and input power.

Results
From Figure 3 it becomes clear that nonlinearities have an average level 10 dB lower than the power generated in the output spectrum by the BLA of the system. Both odd and even nonlinearities are detected, but odd nonlinearities dominate by almost 20 dB Finally, the variation due to stochastic influences is almost 30 dB lower than that due to nonlinear effects.

QUANTIFICATION OF NONLINEAR EFFECTS
Methodology
In order to use the obtained information about nonlinearities from the previous section in controller design, the Higher Order Sinusoidal Input Describing Functions (HOSIDF) of this system are identified. HOSIDFs describe not only the 'direct' response (gain and phase) of a system at the excitation frequency, but describe the response at harmonics of the excitation frequency as well. Consider the following input signal used to identify the HOSIDFs:

$$w(t) = \beta \sin(2\pi f_0^s t). \tag{3}$$

Next, the kth order HOSIDF is defined as:

$$|\mathcal{H}_k(\beta, f_0^s)| = \frac{|Y(kf_0^s)|}{|U(f_0^s)|} \tag{4}$$

$$\angle\mathcal{H}_k(\beta, f_0^s) = \angle Y(kf_0^s) - k\,\angle U(f_0^s), \tag{5}$$

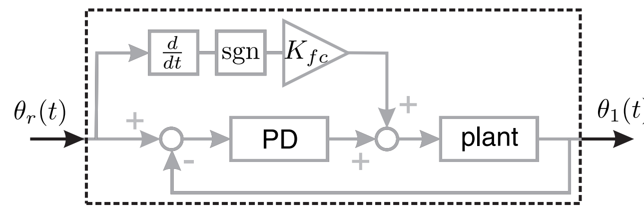

FIGURE 6. Control loop.

where, $Y(kf_0^s) \in \mathbb{C}$ is the output spectrum at the kth harmonic frequency line and $U(f_0^s) \in \mathbb{C}$ the spectral content of the input signal at its fundamental frequency $f_0^s \in \mathbb{R}_{>0}$. HOSIDFs are generally a function of both the input frequency f_0^s and amplitude $\beta \in \mathbb{R}_{>0}$.To assess the quality of the estimates of the HOSIDFs, multiple experiments are conducted and the average of the kth HOSIDF is defined as $\bar{\mathcal{H}}_k$ and its variance as $\varsigma^2_{\bar{\mathcal{H}}_k}$.

Application
The TEM motion stage was excited with frequencies ranging from 5 Hz to 300 Hz in steps of 5 Hz. Each response has been measured 10 times for input powers ranging from 0.07 V to 1.41 V (rms). All measurements have been performed using a SigLab measurement system with a measurement frequency of 5120 Hz and a block length of 2048 points. This results in leakage free measurements since sufficient waiting time is allowed for the transient response to damp out.

Results
A typical series of HOSIDFs is depicted in Figure 5. Odd HOSIDFs are considered, since the even HOSIDFs are very low. The system becomes more linear for increasing value of u_{rms} (decreasing gradient $\partial\mathcal{H}_1/\partial u_{rms}$ and decreasing $|\mathcal{H}_i|,\ i > 1$). The HOSIDFs provide other information about the nonlinear behaviour as well, such as the maximum increase in nonlinear behaviour that can be related to stick-slip transfer [6] and a maximum in nonlinear influences. However, these are outside the scope of this paper.

OPTIMAL FEED FORWARD DESIGN
It was shown that spectral information in the output spectrum can be used to assess the influence of nonlinear phenomena. The next issue addressed in this paper is how to use this information in control, by tuning a feed forward controller based on frequency domain information.

Set-up
The system in Figure 2 is used in feedback with

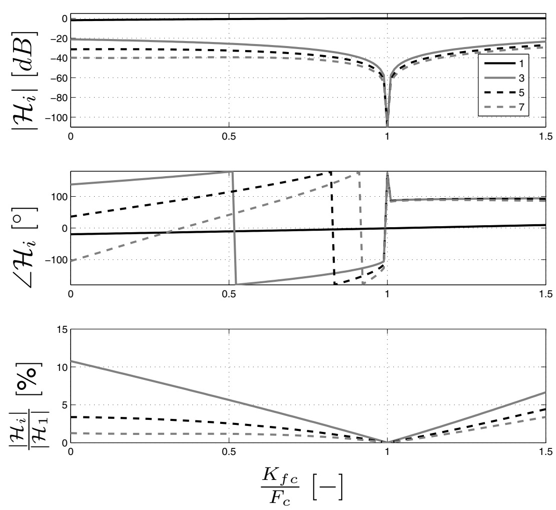

FIGURE 7. Simulation results. (top, center) Magnitude and phase of the first three odd HOSIDFs. (bottom) Ratio between odd nonlinear and linear response components.

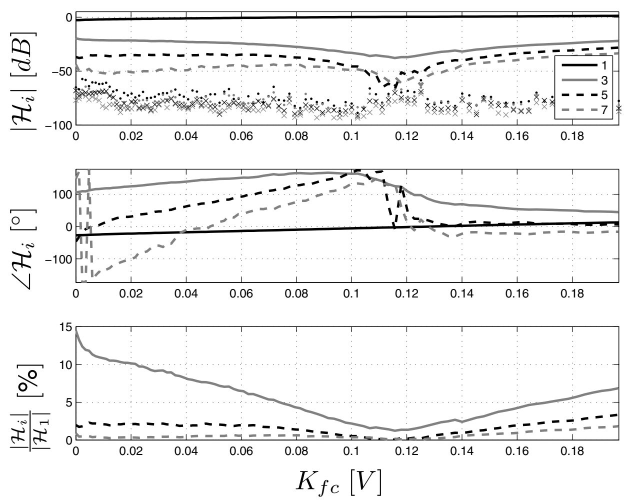

FIGURE 8. Measurement results. (top, center) Magnitude and phase of the first three odd HOSIDFs. (bottom) Ratio between odd nonlinear and linear response components. ($-/--$ average, $\cdot/\times$ standard deviation)

a PD-controller and a reference signal θ_r as depicted in Figure 6. The rotation of the motor $\theta_1(t)$ is the output and controlled variable. Apart from feedback, a nonlinear feed forward is applied to compensate for Coulomb friction. Using a stabilizing PD controller, *the feed forward parameter K_{fc} will be tuned to linearize the input-output behaviour of the closed loop system*. In other words, K_{fc} will be tuned such that the amount of nonlinear (harmonic) content relative to the linear content in the output is minimal, or in terms of the systems HOSIDFs:

$$K^{\star}_{fc} = \underset{K_{fc}\in\mathbb{R}_{\geq 0}}{\operatorname{argmin}} \frac{|\mathcal{H}_i(K_{fc})|}{|\mathcal{H}_1(K_{fc})|} \tag{6}$$

In the sequel, numerical and experimental results are provided that illustrate this tuning method. In both simulations and experiments K_{fc} is varied from 0 (no feed forward) until the system is slightly overcompensated. The HOSIDFs are measured yielding the required minimum. Since the feed forward friction model is a static model, the tuning procedure is performed for one frequency only, assuming Coulomb friction in the plant.

Numerical Results

Figure 7 depicts simulation results of a two mass, spring, damper system similar to the system depicted in Figure 2. The system is subject to Coulomb friction at the motor side and operates in feedback as depicted in Figure 6. From Figure 7 it becomes clear that input-output behaviour of the system becomes more linear with increasing K_{fc} until an optimum is reached when the feed forward equals the Coulomb friction force. Furthermore, the phases of the HOSIDFs turn 180° at the optimal setting.

Experimental Results

Figure 8 shows the same behaviour in experiments using the experimental set-up depicted in Figure 2. An optimum is reached at $K^{\star}_{fc} = 0.1157$ V where the relative level of nonlinearities has decreased from 15% to less than 1.5%. The remaining nonlinear influences are due to effects that are not captured by the feed forward model. Note that even nonlinearites (not depicted) have a relative level of only 4% and are not influenced by the purely odd feed forward.

Discussion

The method presented in this paper enables optimal tuning of (feed forward) parameters in the sense that the input-output behaviour of the system is linearized. The procedure has been demonstrated for Coulomb friction feed forward but may be used to tune arbitrary controllers in the presence of nonlinearities as long as the complete system has a periodic response to a periodic input.

The optimal value $K^{\star}_{fc}$ not necessarily yields the smallest tracking error. Figure 9 shows the response and error observed in experiments: in ab-

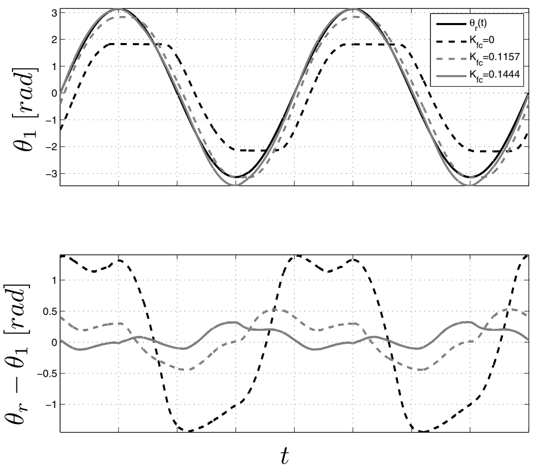

FIGURE 9. Closed loop response with no feed foward ($K_{fc} = 0$), optimal linear response ($K^\star_{fc} = 0.1157$), minimal rms error ($K_{fc} = 1444$). (top) Time response. (bottom) Error.

sence of feed forward ($K_{fc} = 0$), with optimal feed forward ($K_{fc} = K^\star_{fc} = 0.1157$) and at the smallest observed error ($K_{fc} = 0.1444$). It appears that by increasing the feed forward by 25%, the response is still improved. However, for $K_{fc} > K^\star_{fc}$ the feed forward *compensates apples with oranges*. At $K_{fc} = K^\star_{fc}$ the feedforward compenstates nonlinear influences optimally for the chosen feed forward model. Remaining (non)linear error sources are not captured by the feed forward model. For higher K_{fc} the Coulomb feed forward starts to compensates other effects. In this case, the feed forward most likely compensates viscous damping, yielding a local optimum valid only for the type and level of excitation used. Instead, $K^\star_{fc}$ should be selected and a velocity proportional feed forward added. Furthermore, the time varying nature of friction due to wear and tear becomes apparent during longer experiments requiring an adaptive model strucuture for long term compensation.

CONCLUSIONS AND FUTURE RESEARCH

Two methods that allow detection and quantification of nonlinear effects are introduced and used to identify nonlinear influences in an industrial high precision motion stage. Multisine based measurements yield the nature and level of nonlinearities as well as the best linear approximation of the dynamics. Next, nonlinear influences are modeled by measuring the HOSIDFs of the system under test. Finally, the concept behind the HOSIDFs is used to optimally tune a feed forward controller.

The presented method allows tuning of arbitrary controllers linearizing input-output dynamics of systems with a periodic response to a periodic input. It may be used to assess and compare the quality of different controllers and future research aims at optimization algorithms for fast, automated controller tuning. Furthermore, tuning of multiple parameter (non)linear and adaptive controllers is investigated.

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